PSTAT 5A Practice Worksheet 5

Continuous Random Variables and Confidence Intervals

Student Name: ______________________________

2025-07-29

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1 Instructions and Overview

Time Allocation:

- **Intro & Setup** : 10 minutes
- **Section A (Continuous Distributions):** 20 minutes
- **Section B (Confidence Intervals):** 20 minutes
- **Optional Questions:** Do on your own
- **Total:** 50 minutes

Important Instructions:

- Use the formulas and tables provided for guidance
- Round final answers to 4 decimal places unless otherwise specified
- For confidence intervals, always interpret your results in context
- Use z-table or t-table as appropriate
- Show your work for all calculations

Key Formulas Reference:

Continuous Random Variables:

Normal Distribution: $X \sim N(\mu, \sigma^2)$

- **PDF:** $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$
- **Standardization:** $Z = \frac{X-\mu}{\sigma}$ where $Z \sim N(0,1)$
- **Mean:** $E[X] = \mu$
- **Variance:** $\text{Var}(X) = \sigma^2$

Uniform Distribution: $X \sim \text{Uniform}(a,b)$

- **PDF:** $f(x) = \frac{1}{b-a}$ for $a \leq x \leq b$
- **Mean:** $E[X] = \frac{a+b}{2}$
- **Variance:** $\text{Var}(X) = \frac{(b-a)^2}{12}$

Exponential Distribution: $X \sim \text{Exponential}(\lambda)$

- **PDF:** $f(x) = \lambda e^{-\lambda x}$ for $x \geq 0$
- **Mean:** $E[X] = \frac{1}{\lambda}$
- **Variance:** $\text{Var}(X) = \frac{1}{\lambda^2}$

Confidence Intervals:

For Population Mean (known): $\bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$

For Population Mean (unknown): $\bar{x} \pm t_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$

Margin of Error: $E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$ or $E = t_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$

Sample Size: $n = \left(\frac{z_{\alpha/2} \cdot \sigma}{E}\right)^2$

2 Section A: Continuous Random Variables

Estimated time: 20 minutes

Problem A1: Distribution Identification and Properties

For each scenario below, identify the appropriate continuous distribution and find the requested values:

(a) The time (in minutes) between arrivals at a coffee shop follows an exponential distribution with an average of 2 minutes between arrivals.

- What is the parameter λ?
- What is the probability that the next customer arrives within 1 minute?

(b) A random number generator produces values uniformly between 10 and 30.

- What are the parameters a and b?
- What is the expected value and variance?

Work Space:

Problem A2: Normal Distribution Calculations

The heights of adult women in the US are normally distributed with $\mu = 64$ inches and $\sigma = 2.5$ inches.

(a) What is the probability that a randomly selected woman is taller than 67 inches?

(b) What height represents the 25th percentile?

(c) What is the probability that a randomly selected woman has a height between 62 and 68 inches?

> 💡 Tip
>
> **Remember to standardize:** Convert to Z-scores using $Z = \frac{X-\mu}{\sigma}$
> For part (b), you're looking for the value x such that $P(X \leq x) = 0.25$

Work Space:

3 Section B: Confidence Intervals

Estimated time: 20 minutes

Problem B1: Understanding Confidence Intervals

(a) Explain in your own words what a 95% confidence interval means.

(b) A 90% confidence interval for the mean weight of apples is (150g, 170g). What is the sample mean and margin of error?

(c) True or False: "There is a 95% probability that the population mean lies within our calculated 95% confidence interval." Explain your reasoning.

Work Space:

Problem B2: Constructing Confidence Intervals

A sample of 36 students has a mean test score of 78.5 with a standard deviation of 12.

(a) Construct a 95% confidence interval for the population mean test score.

(b) Interpret this interval in the context of the problem.

(c) What would happen to the width of the interval if:

- We increased the confidence level to 99%?
- We increased the sample size to 144?

> 💡 Tip
>
> **Decision Guide:**
> - Use z-distribution when σ is **known** OR $n \geq 30$
> - Use t-distribution when σ is **unknown** AND $n < 30$
> - For 95% CI: $z_{0.025} = 1.96$

Work Space:

4 Optional Questions

Optional Problem: Conceptual Understanding

(a) Explain the key difference between discrete and continuous random variables in terms of:

- The values they can take
- How we calculate probabilities

(b) Why do we use $P(X = x) = 0$ for any specific value x in a continuous distribution?

(c) What's the relationship between PDF and CDF for continuous distributions?

Work Space:

Quick Reference:

Common Z-values:

- 90% CI: $z_{0.05} = 1.645$
- 95% CI: $z_{0.025} = 1.96\$$
- 99% CI: $z_{0.005} = 2.576\$$

Common t-values (selected):

- $df = 24, \alpha = 0.05 : t_{0.025} = 2.064$
- $df = 35, \alpha = 0.05 : t_{0.025} = 2.030$