PSTAT 5A Practice Worksheet 5

Continuous Random Variables and Confidence Intervals

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# 1. Instructions and Overview

**⏰ Time Allocation:**

* **Intro & Setup** : 10 minutes
* **Section A (Continuous Distributions):** 20 minutes
* **Section B (Confidence Intervals):** 20 minutes
* **Optional Questions:** Do on your own
* **Total:** 50 minutes

**📝 Important Instructions:**

* Use the formulas and tables provided for guidance
* Round final answers to 4 decimal places unless otherwise specified
* For confidence intervals, always interpret your results in context
* Use [z-table](https://math.arizona.edu/~rsims/ma464/standardnormaltable.pdf) or [t-table](https://www.sjsu.edu/faculty/gerstman/StatPrimer/t-table.pdf) as appropriate
* Show your work for all calculations

**📚 Key Formulas Reference:**

**Continuous Random Variables:**

**Normal Distribution:** $X∼N\left(μ,σ^{2}\right)$

* **PDF:** $f\left(x\right)=\frac{1}{σ\sqrt{2π}}e^{−\frac{\left(x−μ\right)^{2}}{2σ^{2}}}$
* **Standardization:** $Z=\frac{X−μ}{σ}$ where $Z∼N\left(0,1\right)$
* **Mean:** $E\left[X\right]=μ$
* **Variance:** $Var\left(X\right)=σ^{2}$

**Uniform Distribution:** $X∼Uniform\left(a,b\right)$

* **PDF:** $f\left(x\right)=\frac{1}{b−a}$ for $a\leq x\leq b$
* **Mean:** $E\left[X\right]=\frac{a+b}{2}$
* **Variance:** $Var\left(X\right)=\frac{\left(b−a\right)^{2}}{12}$

**Exponential Distribution:** $X∼Exponential\left(λ\right)$

* **PDF:** $f\left(x\right)=λe^{−λx}$ for $x\geq 0$
* **Mean:** $E\left[X\right]=\frac{1}{λ}$
* **Variance:** $Var\left(X\right)=\frac{1}{λ^{2}}$

**Confidence Intervals:**

**For Population Mean (σ known):** $‾\pm z\_{α/2}⋅\frac{σ}{\sqrt{n}}$

**For Population Mean (σ unknown):** $‾\pm t\_{α/2}⋅\frac{s}{\sqrt{n}}$

**Margin of Error:** $E=z\_{α/2}⋅\frac{σ}{\sqrt{n}}$ or $E=t\_{α/2}⋅\frac{s}{\sqrt{n}}$

**Sample Size:** $n=\left(\frac{z\_{α/2}⋅σ}{E}\right)^{2}$

# 2. Section A: Continuous Random Variables

*⏱️ Estimated time: 20 minutes*

**Problem A1: Distribution Identification and Properties**

For each scenario below, identify the appropriate continuous distribution and find the requested values:

**(a)** The time (in minutes) between arrivals at a coffee shop follows an exponential distribution with an average of 2 minutes between arrivals.

* What is the parameter $λ$?
* What is the probability that the next customer arrives within 1 minute?

**(b)** A random number generator produces values uniformly between 10 and 30.

* What are the parameters a and b?
* What is the expected value and variance?

**Work Space:**

**Problem A2: Normal Distribution Calculations**

The heights of adult women in the US are normally distributed with $μ=64$ inches and $σ=2.5$ inches.

**(a)** What is the probability that a randomly selected woman is taller than $67$ inches?

**(b)** What height represents the $25$th percentile?

**(c)** What is the probability that a randomly selected woman has a height between $62$ and $68$ inches?

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|  Tip |
| **Remember to standardize:** Convert to $Z$-scores using $Z=\frac{X−μ}{σ}$For part (b), you’re looking for the value $x$ such that $P\left(X\leq x\right)=0.25$ |

**Work Space:**

# 3. Section B: Confidence Intervals

*⏱️ Estimated time: 20 minutes*

**Problem B1: Understanding Confidence Intervals**

**(a)** Explain in your own words what a $95\%$ confidence interval means.

**(b)** A $90\%$ confidence interval for the mean weight of apples is (150g, 170g). What is the sample mean and margin of error?

**(c)** True or False: “There is a $95\%$ probability that the population mean lies within our calculated $95\%$ confidence interval.” Explain your reasoning.

**Work Space:**

**Problem B2: Constructing Confidence Intervals**

A sample of $36$ students has a mean test score of $78.5$ with a standard deviation of $12$.

**(a)** Construct a $95\%$ confidence interval for the population mean test score.

**(b)** Interpret this interval in the context of the problem.

**(c)** What would happen to the width of the interval if:

* We increased the confidence level to $99\%$?
* We increased the sample size to $144$?

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|  Tip |
| **Decision Guide:*** Use $z$-distribution when $σ$ is **known** OR $n\geq 30$
* Use $t$-distribution when $σ$ is **unknown** AND $n<30$
* For $95\%$ CI: $z\_{0.025}=1.96$
 |

**Work Space:**

# 4. Optional Questions

**Optional Problem: Conceptual Understanding**

**(a)** Explain the key difference between discrete and continuous random variables in terms of:

* The values they can take
* How we calculate probabilities

**(b)** Why do we use $P\left(X=x\right)=0$ for any specific value $x$ in a continuous distribution?

**(c)** What’s the relationship between PDF and CDF for continuous distributions?

**Work Space:**

**📋 Quick Reference:**

**Common Z-values:**

* $90\%$ CI: $z\_{0.05}=1.645$
* $95\%$ CI: $z\_{0.025}$ = 1.96$
* $99\%$ CI: $z\_{0.005}$ = 2.576$

**Common t-values (selected):**

* $df=24,α=0.05:t\_{0.025}=2.064$
* $df=35,α=0.05:t\_{0.025}=2.030$