PSTAT 5A Practice Worksheet 4

Comprehensive Review: Discrete Random Variables and Distributions

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Table of contents

# 1. Instructions and Overview

**⏰ Time Allocation:**

* **Quiz Review** : 15 minutes
* **Section A (Warm-up):** 15 minutes
* **Section B (Intermediate):** 20 minutes
* **Optional Question:** Do on your own
* **Total:** 50 minutes

**📝 Important Instructions:**

* Use the formulas provided for guidance
* Round final answers to 4 decimal places unless otherwise specified
* Identify the distribution type before calculating
* Show your work for expected value and variance calculations
* Use calculator as needed for factorials and combinations

**📚 Key Formulas Reference:**

**General Random Variable Properties:**

* **Expected Value:** $E\left[X\right]=\sum\_{k}^{​}k⋅P\left(X=k\right)$
* **Variance:** $Var\left(X\right)=E\left[X^{2}\right]−\left(E\left[X\right]\right)^{2}=\sum\_{k}^{​}k^{2}⋅P\left(X=k\right)−μ^{2}$
* **Standard Deviation:** $σ=\sqrt{Var\left(X\right)}$

**Discrete Distributions:**

**Bernoulli Distribution:** $X∼Bernoulli\left(p\right)$

* **PMF:** $P\left(X=k\right)=p^{k}\left(1−p\right)^{1−k}$ for $k\in \{0,1\}$
* **Mean:** $E\left[X\right]=p$
* **Variance:** $Var\left(X\right)=p\left(1−p\right)$

**Binomial Distribution:** $X∼Binomial\left(n,p\right)$

* **PMF:** $P\left(X=k\right)=\left(\genfrac{}{}{0pt}{}{n}{k}\right)p^{k}\left(1−p\right)^{n−k}$ for $k=0,1,2,...,n$
* **Mean:** $E\left[X\right]=np$
* **Variance:** $Var\left(X\right)=np\left(1−p\right)$

**Geometric Distribution:** $X∼Geometric\left(p\right)$

* **PMF:** $P\left(X=k\right)=\left(1−p\right)^{k−1}p$ for $k=1,2,3,...$
* **Mean:** $E\left[X\right]=\frac{1}{p}$
* **Variance:** $Var\left(X\right)=\frac{1−p}{p^{2}}$

**Poisson Distribution:** $X∼Poisson\left(λ\right)$

* **PMF:** $P\left(X=k\right)=\frac{λ^{k}e^{−λ}}{k!}$ for $k=0,1,2,...$
* **Mean:** $E\left[X\right]=λ$
* **Variance:** $Var\left(X\right)=λ$

# 2. Section A: Basic Concepts and Identification

*⏱️ Estimated time: 15 minutes*

**Problem A1: Distribution Identification**

For each scenario below, identify the appropriate discrete distribution and state the parameter(s). **Do not calculate probabilities yet.**

**(a)** A fair coin is flipped until the first head appears. Let X = number of flips needed.

**(b)** A quality control inspector tests 20 randomly selected items from a production line where 5% are defective. Let X = number of defective items found.

**(c)** A website receives visitors at an average rate of 3 per minute. Let X = number of visitors in a 2-minute period.

**(d)** A basketball player shoots one free throw with an 80% success rate. Let X = 1 if successful, 0 if unsuccessful.

**(e)** A student keeps taking a driving test until they pass. The probability of passing on any attempt is 0.7. Let X = number of attempts needed to pass.

**Work Space:**

**Problem A2: Probability Mass Function**

The random variable X has the following probability distribution:

| X | 1 | 2 | 3 | 4 | 5 |
| --- | --- | --- | --- | --- | --- |
| P(X=k) | 0.1 | 0.3 | 0.4 | a | 0.1 |

**(a)** Find the value of $a$.

**(b)** Calculate $P\left(X\leq 3\right)$.

**(c)** Calculate $P\left(X>2\right)$.

**Work Space:**

# 3. Section B: Expected Value and Variance

*⏱️ Estimated time: 20 minutes*

**Problem B1: Manual Calculations**

Using the probability distribution from Problem A2, calculate:

**(a)** The expected value $E\left[X\right]$

**(b)** The variance $Var\left(X\right)$

**(c)** The standard deviation $σ$

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|  Tip |
| **Calculation Strategy:**For expected value: $E\left[X\right]=∑k⋅P\left(X=k\right)$For variance: First find $E\left[X^{2}\right]=∑k^{2}⋅P\left(X=k\right)$, then use $Var\left(X\right)=E\left[X^{2}\right]−\left(E\left[X\right]\right)^{2}$Show your work step by step! |

**Work Space:**

**Problem B2: Bernoulli and Binomial Applications**

A manufacturing process produces items that are defective with probability 0.15.

**(a)** If you select one item randomly, what is the expected value and variance of X = number of defective items?

**(b)** If you select 25 items randomly, what is the expected number of defective items and the standard deviation?

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|  Tip |
| Part (a) is a Bernoulli distribution. Part (b) is a Binomial distribution. Use the formulas from the reference box! |

**Work Space:**

# 4. Optional Questions

**Optional Problem : Conceptual Understanding**

**(a)** Explain the key difference between a Binomial distribution and a Geometric distribution in terms of what they count.

**(b)** When would you use a Poisson distribution instead of a Binomial distribution?

**(c)** If $X∼Binomial\left(n,p\right)$, under what conditions would the variance be maximized?

**Work Space:**