Worksheet 6: Hypothesis Testing Practice

One-Sample Tests & Linear Regression

PSTAT 5A

July 29, 2025

Instructions

Objectives: Practice t-tests, z-tests, and basic linear regression
Guidelines: Show all work, use $\alpha = 0.05$, round to 3 decimal places

Part I: One-Sample t-Test

Quick Reference

Use when: σ unknown **Test statistic:** $t = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}}$ **df:** $n - 1$

Problem 1: Coffee Shop Claims

A coffee shop claims average wait time is 4 minutes. A sample of 8 customers shows wait times:

$$3.2, 4.5, 3.8, 4.1, 3.6, 4.3, 3.9, 4.2$$

Test if the average differs from 4 minutes.

Hypotheses: H_0: _____ H_a: _____
Sample Statistics: $\bar{x} =$ _____ $s =$ _____ $n =$ _____
Test Statistic: $t =$ _____
Critical Value(s): _____ **Decision:** _____

Part II: One-Sample z-Test

Quick Reference

Use when: σ known **Test statistic:** $z = \dfrac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$

Problem 2: Quality Control

A factory produces bolts with target diameter 10.0 mm. Population $\sigma = 0.15$ mm. A sample of 25 bolts has $\bar{x} = 9.94$ mm. Test if the process meets the target.

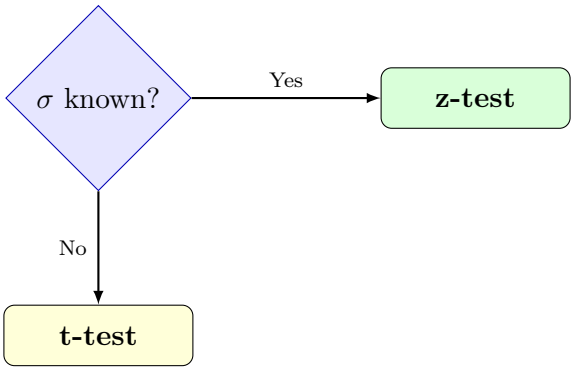
Hypotheses: H_0: _____ H_a: _____

Given: $\mu_0 =$ _____ $\sigma =$ _____ $\bar{x} =$ _____ $n =$ _____

Test Statistic: $z =$ _____

Critical Value(s): _____ **Decision:** _____

When to Use Which Test?



Part III: Linear Regression

A study examines the relationship between study hours (x) and exam scores (y) for 6 students:

Student	1	2	3	4	5	6
Hours (x)	2	4	6	8	10	12
Score (y)	65	70	80	85	90	95

Given summary statistics: $\sum x = 42$, $\sum y = 485$, $\sum x^2 = 364$, $\sum y^2 = 39,975$, $\sum xy = 3,620$

Question 1: Basic Calculations

a) Sample means: $\bar{x} =$ _____ $\bar{y} =$ _____

b) Slope: $b_1 = \frac{\sum xy - n\bar{x}\bar{y}}{\sum x^2 - n\bar{x}^2} =$ _____

c) Y-intercept: $b_0 = \bar{y} - b_1\bar{x} =$ _____

d) Regression equation: $\hat{y} =$ _____

Question 2: Interpretation

a) Predict the exam score for a student who studies 7 hours:

b) Interpret the slope in context:

Question 3: Correlation

a) Calculate correlation: $r = \frac{\sum xy - n\bar{x}\bar{y}}{\sqrt{(\sum x^2 - n\bar{x}^2)(\sum y^2 - n\bar{y}^2)}} =$ _____

b) Calculate $r^2 =$ _____ and interpret:

Answer Key

Part I: t-Test Solution

Problem 1:

- $H_0 : \mu = 4$, $H_a : \mu \neq 4$
- $\bar{x} = 3.95$, $s = 0.338$, $n = 8$
- $t = \frac{3.95-4}{0.338/\sqrt{8}} = -0.419$
- Critical values: ± 2.365 (df $= 7$)
- Decision: Fail to reject H_0 (no significant difference)

Part II: z-Test Solution

Problem 2:

- $H_0 : \mu = 10.0$, $H_a : \mu \neq 10.0$
- $z = \frac{9.94-10.0}{0.15/\sqrt{25}} = -2.0$
- Critical values: ± 1.96
- Decision: Reject H_0 (process not meeting target)

Part III: Linear Regression Solutions

Basic Calculations:

- $\bar{x} = 7$, $\bar{y} = 80.833$
- $b_1 = 2.5$, $b_0 = 63.333$
- Regression: $\hat{y} = 63.333 + 2.5x$

Interpretations:

- 7 hours prediction: $\hat{y} = 63.333 + 2.5(7) = 80.833$
- Slope: Each additional study hour increases exam score by 2.5 points
- $r = 0.986$, $r^2 = 0.971$ (97.1% of variation explained)

Quick Formulas

t-test: $t = \frac{\bar{x}-\mu_0}{s/\sqrt{n}}$ **z-test:** $z = \frac{\bar{x}-\mu_0}{\sigma/\sqrt{n}}$

Regression: $b_1 = \frac{\sum xy - n\bar{x}\bar{y}}{\sum x^2 - n\bar{x}^2}$ $b_0 = \bar{y} - b_1\bar{x}$

Correlation: $r = \frac{\sum xy - n\bar{x}\bar{y}}{\sqrt{(\sum x^2 - n\bar{x}^2)(\sum y^2 - n\bar{y}^2)}}$