PSTAT 5A Practice Worksheet 3

Comprehensive Review: Probability, Counting, an Conditional Probability

Student Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

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# 1. Instructions and Overview

**⏰ Time Allocation:**

* **Section A (Warm-up):** 8 minutes
* **Section B (Intermediate):** 15 minutes
* **Section C (Advanced):** 15 minutes
* **Section D (Review):** 12 minutes
* **Total:** 50 minutes

**📝 Important Instructions:**

* Use the formulas provided for guidance
* Round final answers to 4 decimal places unless otherwise specified
* Identify your approach before calculating
* Use calculator as needed

**📚 Key Formulas Reference:**

**Basic Probability:**

* **Conditional Probability:** $P\left(A|B\right)=\frac{P\left(A∩B\right)}{P\left(B\right)}$
* **Bayes’ Theorem:** $P\left(A|B\right)=\frac{P\left(B|A\right)⋅P\left(A\right)}{P\left(B\right)}$
* **Law of Total Probability:** $P\left(A\right)=∑P\left(A|B\_{i}\right)⋅P\left(B\_{i}\right)$
* **Addition Rule:** $P\left(A∪B\right)=P\left(A\right)+P\left(B\right)−P\left(A∩B\right)$
* **Multiplication Rule:** $P\left(A∩B\right)=P\left(A\right)⋅P\left(B|A\right)=P\left(B\right)⋅P\left(A|B\right)$

**Counting:**

* **Permutations:** $P\left(n,r\right)=\frac{n!}{\left(n−r\right)!}$
* **Combinations:** $C\left(n,r\right)=\left(\genfrac{}{}{0pt}{}{n}{r}\right)=\frac{n!}{r!\left(n−r\right)!}$

# 2. Section A: Probability

*⏱️ Estimated time: 8 minutes*

**Problem A1: Probability Distributions**

Each row in the table below is a proposed grade distribution for a class. Identify each as a valid or invalid probability distribution, and explain your reasoning.

| Class | A | B | C | D | F |
| --- | --- | --- | --- | --- | --- |
| (a) | 0.3 | 0.3 | 0.3 | 0.2 | 0.1 |
| (b) | 0 | 0 | 1 | 0 | 0 |
| (c) | 0.3 | 0.3 | 0.3 | 0 | 0 |
| (d) | 0.3 | 0.5 | 0.2 | 0.1 | -0.1 |
| (e) | 0.2 | 0.4 | 0.2 | 0.1 | 0.1 |
| (f) | 0 | -0.1 | 1.1 | 0 | 0 |

**Work Space:**

# 3. Section B: Permutations and Combination

*⏱️ Estimated time: 15 minutes*

**Problem B1: Permutations and Combinations**

A cybersecurity team needs to create a secure access protocol.

**Part (a):** How many 6-character passwords can be formed using 3 specific letters and 3 specific digits if repetitions are not allowed and letters must come before digits?

|  |
| --- |
|  Tip |
| Since letters must come before digits, think of this as two separate arrangement problems:* First, arrange the 3 letters in the first 3 positions
* Then, arrange the 3 digits in the last 3 positions
* Use the multiplication principle to combine these results
 |

**Part (b):** If the team wants to select 4 people from 12 employees to form a security committee where order doesn’t matter, how many ways can this be done?

|  |
| --- |
|  Tip |
| Since order doesn’t matter, this is a combination problem. Ask yourself:* Are we arranging people in specific positions, or just selecting a group?
* Which formula should you use: $P\left(n,r\right)$ or $C\left(n,r\right)$?
 |

**Work Space:**

# 4. Section C: Conditional Probability

*⏱️ Estimated time: 15 minutes*

**Problem B1: Conditional Probability and Medical Testing**

A new COVID variant test has the following characteristics:

* The variant affects 3% of the tested population
* The test correctly identifies 95% of people with the variant (sensitivity)
* The test correctly identifies 92% of people without the variant (specificity)

**Part (a):** What is the probability that a randomly selected person tests positive?

**Part (b):** If someone tests positive, what is the probability they actually have the variant?

**Part (c):** If someone tests negative, what is the probability they actually don’t have the variant?

**Part (d) [Challenge]:** The health department wants to reduce false positives. They decide to require two consecutive positive tests for a positive diagnosis. Assuming test results are independent, what is the new probability that someone with two positive tests actually has the variant?

**Work Space:**

# 5. Section C: Conditional Probability

*⏱️ Estimated time: 15 minutes*

**Problem C1: Advanced Counting with Restrictions**

A restaurant offers a prix fixe menu where customers must choose:

* 1 appetizer from 6 options
* 1 main course from 8 options
* 1 dessert from 5 options

However, there are restrictions:

* If you choose the seafood appetizer, you cannot choose the vegetarian main course
* If you choose the chocolate dessert, you must choose either the beef or chicken main course (3 of the 8 main courses)

**Part (a):** How many valid meal combinations are possible?

**Part (b):** If customers choose randomly among valid combinations, what is the probability someone chooses the chocolate dessert?

**Work Space:**

# 6. Section D: Review

*⏱️ Estimated time: 12 minutes*

**Problem B3: Daily Expenses**

Sally gets a cup of coffee and a muffin every day for breakfast from one of the many coffee shops in her neighborhood. She picks a coffee shop each morning at random and independently of previous days. The average price of a cup of coffee is $1.40 with a standard deviation of 30¢ ($0.30), the average price of a muffin is $2.50 with a standard deviation of 15¢, and the two prices are independent of each other.

**Part (a):** What is the mean and standard deviation of the amount she spends on breakfast daily?

**Part (b):** What is the mean and standard deviation of the amount she spends on breakfast weekly (7 days)?

**Work Space:**